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# Compulsory Deep Mixing of $^3\text{He}$ and CNO Isotopes on the First Giant Branch

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**Abstract.** We have found a deep-mixing process which occurs during First Giant Branch (FGB) evolution. It begins at the point in evolution where the surface convection zone (SCZ), having previously grown in size, starts to shrink, and it is driven by a local minimum that develops in the mean molecular weight as a result of the burning of  $^3\text{He}$ . This mixing can solve two important observational problems. One is why the interstellar medium (ISM) has not been considerably enriched in  $^3\text{He}$  since the Big Bang. The other is why products of nucleosynthesis such as  $^{13}\text{C}$  are progressively enriched on the upper FGB, when classical stellar modeling says that no further enrichment should take beyond the First Dredge-Up (FDU) episode, somewhat below the middle of the FGB.

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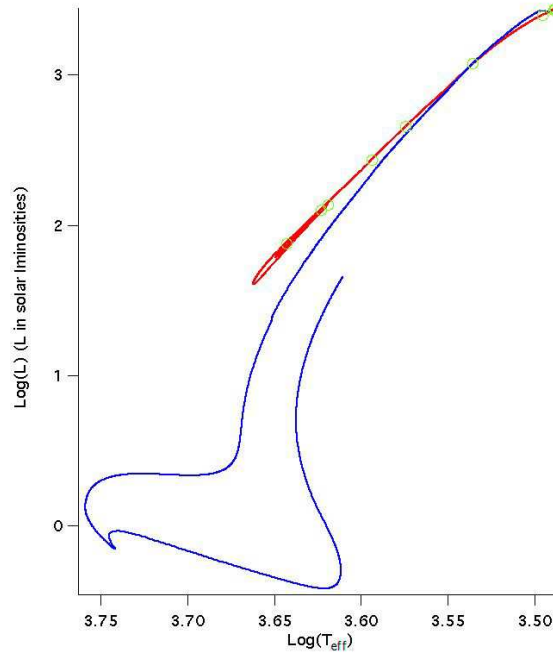
## INTRODUCTION

We describe a serendipitous discovery made during 3D modeling of the He flash. The discovery provides, we believe, an explanation for two important problems that have been noted in the study of the evolution of low-mass stars over the last 30 years or so. One is a problem regarding the cosmic abundance of  $^3\text{He}$ , which appears to be less abundant than we would expect on the basis that low-mass stars are prolific sources of  $^3\text{He}$ . The other is that there is found to be an increase of  $^{13}\text{C}$  relative to  $^{12}\text{C}$  on the FGB, well beyond the point where stellar models have suggested that the increase should stop.

The Big Bang is supposed to produce  $^3\text{He}$  at  $\sim 10^{-5}$  by mass fraction. Low-mass stars ( $1 - 2 M_{\odot}$ ) produce further  $^3\text{He}$ ,  $\sim 10^{-3}$ , in their interiors, and mix it into their convective envelopes on the FGB. Later they eject  $\gtrsim 0.3 M_{\odot}$  of envelope into the ISM further up on the FGB. So the ISM ought to be richer in  $^3\text{He}$  than the Big-Bang value, but it is not (Hata et al. 1995).

## CONVENTIONAL EVOLUTION

Fig. 1 shows the evolutionary track of a Pop I star of  $1 M_{\odot}$ . It starts as a pre-Main-Sequence star near the middle of the diagram and evolves towards the Zero-Age Main Sequence (ZAMS) in the bottom left-hand corner. Hydrogen burning moves it upwards across the Main Sequence (MS) and then briefly horizontally, to the base of the First Giant Branch (FGB). It is developing a deep Surface Convective Zone (SCZ) at this point, and the envelope continues to deepen as the star climbs to a luminosity of about



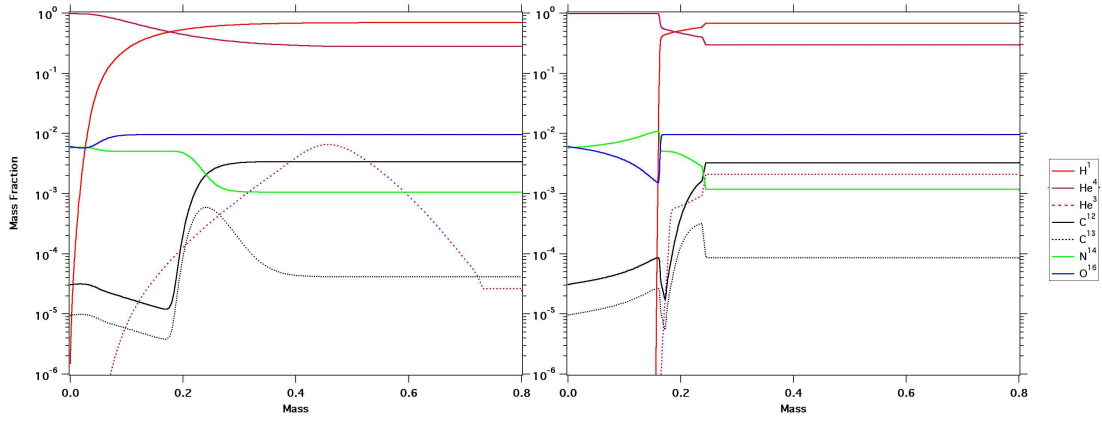
**FIGURE 1.** Hertzsprung-Russell diagram of a  $1 M_{\odot}$  star approaching the Main Sequence (MS), leaving it again to climb the First Giant Branch (FGB), passing through the helium flash, reaching down to the Horizontal Branch (HB) and ending near the top of the Asymptotic Giant Branch (AGB).

$100 L_{\odot}$ . During the later part of this deepening, the envelope entrains material that, at an earlier stage on the MS, underwent nuclear processing. Thus the convective envelope, and importantly the visible photosphere, should become enriched in various nuclei, in particular  $^3\text{He}$  and  $^{13}\text{C}$ .

Beyond about the middle of the FGB, the base of the SCZ starts moving outwards again, and as a consequence we expect no further nuclear enrichment. The hydrogen-burning shell moves outwards (in mass), and the helium core mass, and in addition its temperature, grows. This slow evolution accelerates very rapidly once the core temperature is hot enough to burn  $^4\text{He}$  to  $^{12}\text{C}$ , the Helium Flash. The model was evolved through this and then down to the Horizontal Branch (HB), the minimum (in luminosity) seen at about  $50 L_{\odot}$ . It was further followed back up the Second, or Asymptotic, Giant Branch (AGB), but this later evolution is not relevant to the present discussion.

Observed HB stars are usually considerably hotter than the one shown here. This is normally attributed to progressive mass loss by stellar wind on the FGB. Such mass loss has only a minor effect on the FGB, but it allows HB stars to be considerably hotter ( $\log T \gtrsim 4.0$ ). Such mass loss was not however included in the model of Fig. 1.

Fig. 2 shows the abundances of several elements through a similar star (but  $0.8 M_{\odot}$ ), at two different stages. The first (at the left) is at the end of the MS, and the second is at the point on the FGB where the SCZ is deepest. We see that during the MS a considerable amount of  $^3\text{He}$  has been produced; the initial value was the value still to be seen at the surface ( $\sim 2.5 \cdot 10^{-5}$ ), and the maximum value in the interior is about 300 times that. Once the SCZ has bottomed out, in the right-hand panel, the surface value of  $^3\text{He}$  has



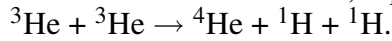
**FIGURE 2.** Left panel: abundances at the end of the MS, as a function of mass coordinate. Notice the  $^3\text{He}$  peak (red dots) and the  $^{13}\text{C}$  peak (black dots). Right panel: halfway up the FGB, the surface convection zone (SCZ) reaches its deepest extent. The surface  $^3\text{He}$  is increased by  $\sim 100$ ,  $^{13}\text{C}$  by  $\sim 3$ .

been increased from the initial value to  $\sim 2 \cdot 10^{-3}$ , ie. by a factor of 80. At the same time, and for much the same reason, the surface abundance of  $^{13}\text{C}$  has increased from about  $4 \cdot 10^{-5}$  to about  $9 \cdot 10^{-5}$ .

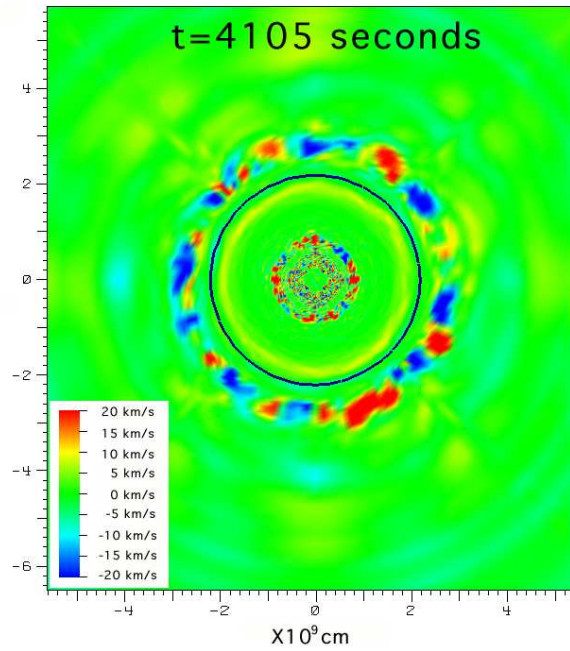
Since FGB stars lose substantial mass by stellar wind before settling down as HB stars, we expect the ISM to be substantially enriched in  $^3\text{He}$  by low-mass stars. Yet the amount of  $^3\text{He}$  observed in the ISM is little different from what is predicted from Big Bang nucleosynthesis (Hata et al. 1995).

Fig. 3 shows a still from a movie of a 3D simulation of the helium flash (Dearborn et al. 2006). The dark solid ring is the hydrogen-burning shell. Fairly far inside that ring can be seen some turbulent motion, which is the beginning of turbulent convection driven by the intense helium-burning. But the important point for the present discussion is a ring of apparently turbulent motion just *outside* the hydrogen burning shell. Note that the base of the conventional SCZ is well outside the boundary of this frame; and also that some slight circular arcs of motion exist which are the remains of transient spherical pressure waves generated by the fact that the initial model of the 3D run was not in exact 3D hydrostatic equilibrium.

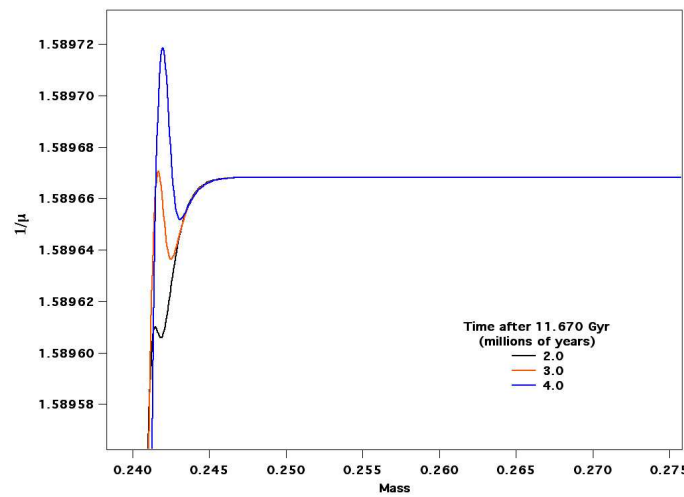
The outer ring of turbulent motion puzzled us. Was it some numerical artefact? A very close inspection of the model, however, revealed a slight local minimum in molecular weight in the 1D hydrostatic model from which the 3D model was started, and further inspection of earlier 1D models showed that this was a legitimate result of nuclear evolution, specifically of the reaction



A relatively high abundance of  $^3\text{He}$  is left behind as the SCZ retreats, and this burns by the above reaction in a temperature shell that is well below the SCZ yet fairly far above the hydrogen-burning shell. This reaction *lowers* the mean molecular weight  $\mu$ , unusually for a stellar nuclear reaction. Although first noticed during our helium-flash calculation, the phenomenon shows up earlier, as soon as the classical SCZ, having penetrated to maximum depth inwards, starts moving outwards again.



**FIGURE 3.** A 3D model of the He flash, in 2D cross-section. The dark blue circle is the H-burning shell. The SCZ is far outside frame. Turbulent motion occurs in the He-burning shell very near the centre, but also in a shell outside but near the H-burning shell.



**FIGURE 4.** Growing peak in  $1/\mu$

## $\Delta\mu$ -MIXING

We went back down the 1D Giant Branch to a point not long after the SCZ bottomed out. Fig. 4 illustrates three early stages in the formation of the  $\mu$ -inversion: note that it is  $1/\mu$  that is plotted here.

We mapped a 1D model that had developed this  $\mu$ -inversion on to a 3D model, and evolved the latter using the 3D hydrodynamics code ‘Djehuty’ of the Lawrence Livermore National Laboratory. The code is discussed in some detail by Dearborn, Lattanzio & Eggleton (2006). This code can in principle handle a complete star; but to economise on meshpoints we placed our outer boundary just below the SCZ. When we started our 3D simulation from the third of the models in Fig. 4, we found that it was exactly at this  $\mu$  inversion that the turbulent motion was initiated. We used  $\sim 10^7$  meshpoints. The timestep is Courant-limited, to about 0.1 sec, and we ran it on 351 processors for  $\sim 10,000$  secs, taking a few days of cpu time.

Although we believe it is unequivocal that some turbulent motion will be driven by this inversion, we do not claim that our numerical model reproduces this motion exactly. We do not have the numerical resolution to examine it in detail, nor do we have the cpu time to follow it for years of star time rather than hours. Nevertheless we feel confident that the  $\Delta\mu$ -mixing region must grow with time until it reaches out to and unites with the normal SCZ, and that it will last as long as there is enough  $^3\text{He}$  left to fuel the  $^3\text{He}$ -burning,  $\mu$ -inverted shell.

We estimate that mixing driven by this instability (which we refer to as ‘ $\Delta\mu$ -mixing’) will allow most of the  $^3\text{He}$  produced during the MS phase to be destroyed again, on the upper half of the FGB. This solves the problem mentioned earlier. At the same time, the mixing will also enhance the abundance of  $^{13}\text{C}$  relative to  $^{12}\text{C}$  on the second half of the FGB.

Two ‘frequently-asked questions’ are:

(i) Doesn’t the motion due to the instability soon stop, as the molecular-weight inversion gets diluted by the mixing it causes?

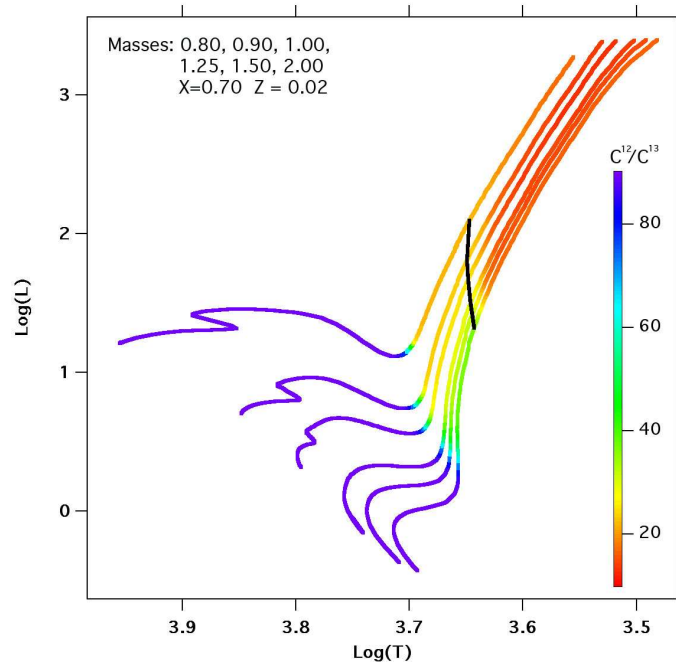
*Ans.* No, because the mixing advects fresh  $^3\text{He}$  into the  $^3\text{He}$ -burning zone at much the same rate as it advects the products out. We conclude that the  $\Delta\mu$ -mixing zone will keep growing, until it unites with the normal SCZ.

(ii) Shouldn’t the motion be very small compared to the motion in the normal SCZ, because  $\Delta\mu/\mu$  is small ( $\sim 10^{-4}$ )?

*Ans.* No, because the buoyancy term that drives the normal convection,  $\Delta T/T$ , is also small.  $\Delta T$  is the *excess* temperature of an element that has been raised adiabatically (by about one pressure scaleheight) over the ambient temperature. Since, in the deeper parts of an SCZ, the ambient temperature gradient is very nearly adiabatic,  $\Delta T/T$  is quite small ( $\sim 10^{-4}$ !).

Our 3D model produced rapid motion, comparable to the normal convective motion, because the motion generated by the  $\Delta\mu$ -instability was artificially suppressed in the 1D model, allowing the  $\mu$ -inversion to reach its maximum size. In reality, motion will start sooner, almost as soon as the  $\mu$ -inversion begins. The actual velocities produced will be smaller, but we estimate that they will still be quite enough to mix the entire zone between the  $^3\text{He}$ -burning shell and the classical SCZ. The velocities would have to be less than  $\sim 10^{-4}$  cm/s in order *not* to do this.

By various back-of-the-envelope calculations we estimate that the velocities are likely to be in the range 0.2 – 2 cm/s. Our 3D model, starting from an artificially large inversion, produced velocities of  $\sim 200$  m/s. We have modeled in 1D a number of low-mass stars, incorporating a diffusion-equation algorithm that we hope captures the essence of the  $\Delta\mu$ -mixing, if not the details.



**FIGURE 5.** Evolutionary tracks on the Hertzsprung-Russell diagram for a variety of masses from 0.8 to  $2.0 M_{\odot}$ . The tracks are colour-coded for the ratio  $^{12}\text{C}/^{13}\text{C}$ ; blue is the ZAMS value (90), and the deepest red is a value of 10. The black line indicates where the SCZ is deepest; with our  $\Delta\mu$ -mixing there would be no further change in  $^{12}\text{C}/^{13}\text{C}$  above this line.

Over the last 30 years or so, observation has suggested that some mixing process must connect the SCZs of FGB stars to the H-burning shell, or at least to a region close enough to the main shell that some CNO abundances can be changed. Giants often show  $^{13}\text{C}$  abundances elevated above the value expected at and beyond the first dredge-up (Suntzeff 1993). Also  $^{14}\text{N}/^{12}\text{C}$  is often elevated. We expect the SCZ to be processed to a substantially deeper level, to at least the  $^3\text{He}$ -burning shell, where some at least of the  $^{12}\text{C}$  can burn to  $^{13}\text{C}$ ; and perhaps there is sufficient ‘undershooting’ to process significant amounts of  $^{12}\text{C}$  to  $^{14}\text{N}$ .

We have incorporated into our 1D model a crude diffusive-mixing model where the diffusion coefficient is proportional to the difference between the local  $\mu$  and the minimal  $\mu$ , provided that the minimal  $\mu$  is further in. The magnitude of the mixing rate that we used was chosen to give roughly the velocity of rising material that we expect between the  $\mu$ -inversion peak and the SCZ. The results, fortunately but not surprisingly, do not depend very strongly on the value of mixing rate chosen, within a wide but we believe realistic range.

Fig. 5 shows the results we obtained, for a range of masses of Pop. I stars. A black nearly-vertical line near the centre of the Figure indicates where the SCZ reached its deepest extent. In conventional models there would be no change in the  $^{12}\text{C}/^{13}\text{C}$  ratio, typically about 25 – 30, beyond this point. But our  $\Delta\mu$ -mixing process allows the ratio to be reduced progressively further, to about 10 in all but the most massive model.



## CONCLUSIONS

We feel that we can draw three conclusions:

- (1) Interesting processes are often discovered by accident, while one is looking for something else. So much for those who feel that a project should identify what it will deliver in each of three successive years.
- (2) 3D computation, though expensive, can produce something useful and unexpected. With the aid of a short run in 3D, one can hope to refine a simplistic 1D model which will obtain long-term results cheaply.
- (3) It is possible to understand the absence of substantial  $^3\text{He}$  enrichment in the Galaxy, and also the observed progressive enhancement of  $^{13}\text{C}$  relative to  $^{12}\text{C}$  on the upper part of the FGB, in terms of an unavoidable deep-mixing process driven by a molecular-weight inversion that is itself caused by  $^3\text{He}$ -burning on the FGB.

## ACKNOWLEDGMENTS

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